A new Constraint Programming model for the Multiple Constant Multiplication

ModRef Workshop - 31th International Conference on Principles and Practice of CP

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August 11, 2025

Introduction

Introduction

- Embedded computations everywhere
- Arithmetic operators tailored to applications' requirements

Multiple Constant Multiplication problem (MCM)

- Multiplying an integer by several fixed integer target constants
- Exploiting *a priori* knowledge of constants to design optimized implementations
- Avoiding costly generic multiplier

MCM is used in:

- Digital Signal Processing for linear digital filter evaluation [Aksoy, 2011; Garcia, 2022; Howard, 2014; Kumm, 2023; Meher, 2011]
- Discrete Transforms (Discrete Cosine, Fast Fourier) [Ghissoni, 2010; Wendt, 2015]
- Inference of Deep Neural Networks [Garcia, 2025; Habermann, 2022]

Motivation - Single Constant Multiplication

$$X = \boxed{X_6}\boxed{X_5}\boxed{X_4}\boxed{X_3}\boxed{X_2}\boxed{X_1}\boxed{X_0}$$

$$\times\ 93 = \boxed{1}\boxed{0}\boxed{1}\boxed{1}\boxed{1}\boxed{0}\boxed{1}$$

$$= \quad 1 \times \boxed{X_6}\boxed{X_5}\boxed{X_4}\boxed{X_3}\boxed{X_2}\boxed{X_1}\boxed{X_0}$$

$$\cancel{+\ 0 \times \boxed{X_6}\boxed{X_5}\boxed{X_4}\boxed{X_3}\boxed{X_2}\boxed{X_1}\boxed{X_0}}$$

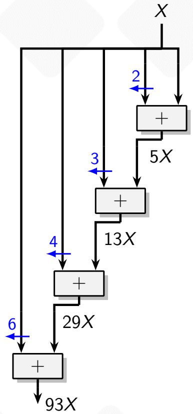
$$+\ 1 \times \boxed{X_6}\boxed{X_5}\boxed{X_4}\boxed{X_3}\boxed{X_2}\boxed{X_1}\boxed{X_0} \xleftarrow{\ 2\ }$$

$$+\ 1 \times \boxed{X_6}\boxed{X_5}\boxed{X_4}\boxed{X_3}\boxed{X_2}\boxed{X_1}\boxed{X_0} \xleftarrow{\ 3\ }$$

$$+\ 1 \times \boxed{X_6}\boxed{X_5}\boxed{X_4}\boxed{X_3}\boxed{X_2}\boxed{X_1}\boxed{X_0} \xleftarrow{\ 4\ }$$

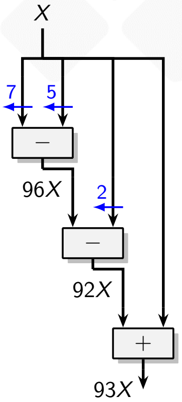
$$\cancel{+\ 0 \times \boxed{X_6}\boxed{X_5}\boxed{X_4}\boxed{X_3}\boxed{X_2}\boxed{X_1}\boxed{X_0}}$$

$$+\ 1 \times \boxed{X_6}\boxed{X_5}\boxed{X_4}\boxed{X_3}\boxed{X_2}\boxed{X_1}\boxed{X_0} \xleftarrow{\ 6\ }$$



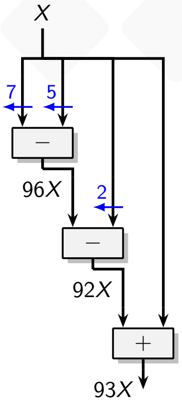
Binary approach for $T = \{93\}$:
$93_{dec} = 1011101_{bin}$

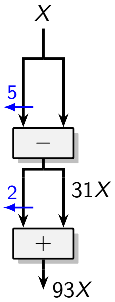
Motivation - Single Constant Multiplication



Canonical Sign Digit (CSD) approach for $T = \{93\}$:
$93_{dec} = 10\bar{1}00\bar{1}01_{CSD}$

Motivation - Single Constant Multiplication





Canonical Sign Digit (CSD) approach for $T = \{93\}$:
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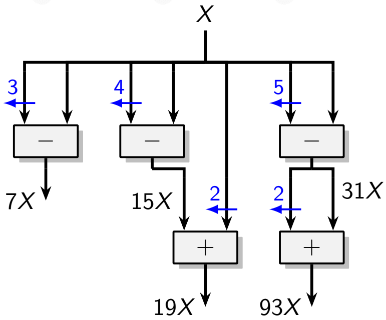
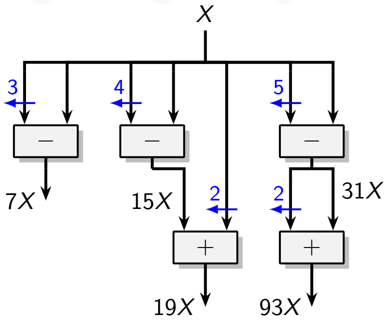
Optimal graph for $T = \{93\}$

Motivation - Multiple Constant Multiplication



(a) Single constant approach for $T = \{7, 19, 93\}$

Motivation - Multiple Constant Multiplication



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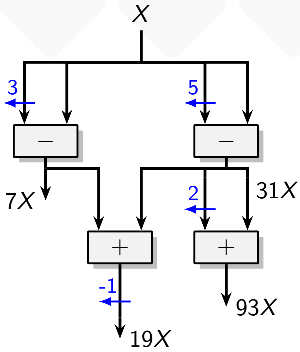

(b) Optimal graph for $T = \{7, 19, 93\}$

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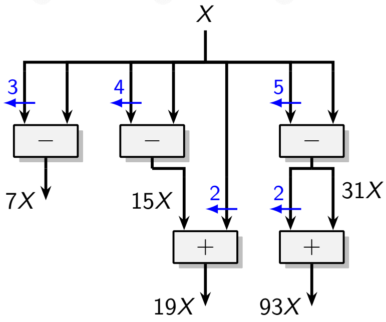

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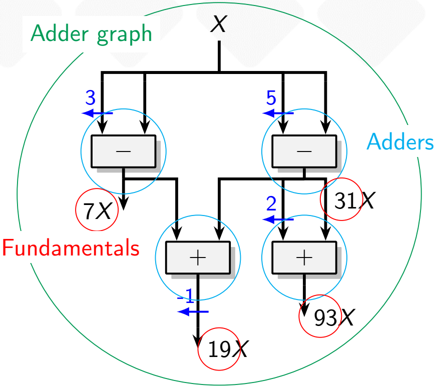

(b) Optimal graph for $T = \{7, 19, 93\}$

Formal definition - MCM problem

- Optimization metric: number of adders
- ⚠ Doesn't perfectly reflect hardware cost but good proxy variable

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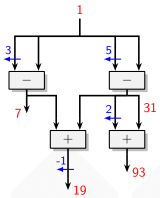
Definition (MCM-Adders)

Problem: MCM-Adders(T, w)

Input: *Target constant set* T*,*
word-length w

Output: *MCM implementation producing* T *with fundamentals written on* w *bits minimizing adders number*

Input:
$T = \{7, 19, 93\}$,
$w = 5$
Output:



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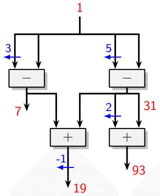
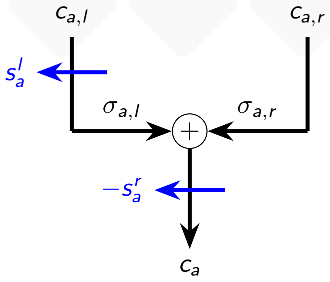


Exact and declarative approaches:

- Already modeled with ILP [Kumm, 2018; Garcia, 2023]
- Already modeled with SAT [Fiege, 2024]
- Waiting to be modeled with CP!

CP Model

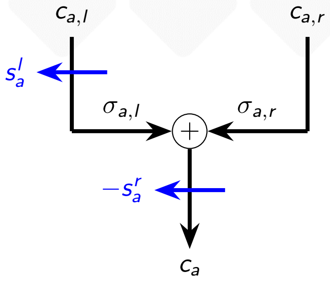
Constraints - Adder modeling



$$c_a = 2^{-s_a^r}((-1)^{\sigma_{a,l}} 2^{s_a^l} c_{a,l} + (-1)^{\sigma_{a,r}} c_{a,r})$$

Left shift only on the left input or (exclusive) right shift on both inputs [Dempster, 1994]

Constraints - Adder modeling



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ILP: $$\underbrace{2^{s_a^r} c_a}_{c_a^{nsh}} = \underbrace{(-1)^{\sigma_{a,l}} \underbrace{2^{s_a^l} c_{a,l}}_{c_{a,l}^{sh}}}_{c_{a,l}^{sh,sg}} + \underbrace{(-1)^{\sigma_{a,r}} \underbrace{c_{a,r}}_{c_{a,r}^{sh}}}_{c_{a,r}^{sh,sg}}$$

$$\begin{cases} c_a^{nsh} = 2^s c_a \text{ if } \Psi_{a,s} = 1 & \text{w+1 var} \quad \text{2w+1 cstr} \\ c_{a,l}^{sh} = 2^s c_{a,l} \text{ if } \Phi_{a,s} = 1 & \text{w+1 var} \quad \text{2w+1 cstr} \\ c_{a,i}^{sh,sg} = -c_{a,i}^{sh} \text{ iff } \sigma_{a,i} = 1 & \text{4 var} \quad \text{4 cstr} \end{cases}$$

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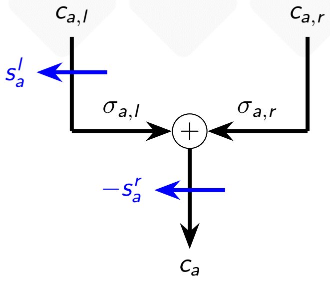

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Left shift only on the left input or (exclusive) right shift on both inputs [Dempster, 1994]

$$\text{ILP:}\quad \underbrace{2^{s_a^r} c_a}_{c_a^{nsh}} = \underbrace{(-1)^{\sigma_{a,l}} \underbrace{2^{s_a^l} c_{a,l}}_{c_{a,l}^{sh}}}_{c_{a,l}^{sh,sg}} + \underbrace{(-1)^{\sigma_{a,r}} \underbrace{c_{a,r}}_{c_{a,r}^{sh}}}_{c_{a,r}^{sh,sg}}$$

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$$\text{CP:}\quad \underbrace{\underbrace{2^{s_a^r}}_{k_a} c_a}_{c_a^{nsh}} = \underbrace{\underbrace{(-1)^{\sigma_{a,l}} 2^{s_a^l}}_{k_{a,l}} c_{a,l}}_{c_{a,l}^{sh,sg}} + \underbrace{\underbrace{(-1)^{\sigma_{a,r}}}_{k_{a,r}} c_{a,r}}_{c_{a,r}^{sh,sg}}$$

$$\begin{cases} c_a^{nsh} = k_a \times c_a & \text{2 var} & \text{1 cstr} \\ c_{a,i}^{sh,sg} = k_{a,i} \times c_{a,i} & \text{4 var} & \text{2 cstr} \end{cases}$$

Constraints - Adder modeling

$$\underbrace{\underbrace{2^{s_a^r}}_{k_a} c_a}_{c_a^{nsh}} = \underbrace{\underbrace{(-1)^{\sigma_{a,l}} 2^{s_a^l}}_{k_{a,l}} c_{a,l}}_{c_{a,l}^{sh,sg}} + \underbrace{\underbrace{(-1)^{\sigma_{a,r}}}_{k_{a,r}} c_{a,r}}_{c_{a,r}^{sh,sg}}$$

Variables:

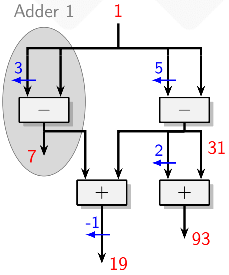
- $c_a \in [\![1, 2^w]\!]$: fundamental produced by a
- $c_{a,l},\ c_{a,r} \in [\![1, 2^w]\!]$: left and right inputs
- $k_a \in \{2^s \mid s \in [\![0, w]\!]\}$
- $k_{a,l} \in \{-2^s, 2^s \mid s \in [\![0, w]\!]\}$, $k_{a,r} \in \{-1, 1\}$
- $c_a^{nsh} \in [\![0, 2^{w+1}]\!]$
- $c_{a,l}^{sh,sg} \in [\![-2^{w+1}, 2^{w+1}]\!]$, $c_{a,r}^{sh,sg} \in [\![-2^w, 2^w]\!]$

Constraints:

C1: $k_a \times c_a = c_a^{nsh}$

C2: $k_{a,l} \times c_{a,l} = c_{a,l}^{sh,sg}$ and $k_{a,r} \times c_{a,r} = c_{a,r}^{sh,sg}$

C3: $c_{a,l}^{sh,sg} + c_{a,r}^{sh,sg} = c_a^{nsh}$



Adder 1:

- $k_1 \times c_1 = k_{1,l} \times c_{1,l} + k_{1,r} \times c_{1,r}$
  $2^0 \times 7 = (-1)^0 2^3 \times 1 + (-1)^1 \times 1$

Constraints - MCM consistency

Variables:

- $ind_{a,l},\ ind_{a,r} \in [\![0, a-1]\!]$: index of left or right input of adder a

$ind_{a,i} < a \Rightarrow$ avoiding cycles

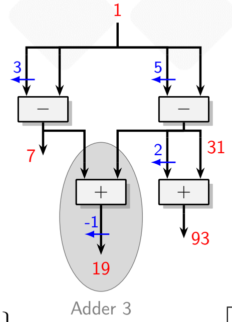
Constraints:

C4: $c_{a,l} = c_{ind_{a,l}} \ \wedge\ c_{a,r} = c_{ind_{a,r}}$

C5: $T \subseteq \{c_a \mid a \in [\![1, N]\!]\}$

C6: $c_0 = 1$

C7: $\text{allDifferent}((c_a)_{a \in [\![0,N]\!]})$



- $T \subseteq \{c_1, c_2, c_3\}$
  $\{7, 19, 93\} \subseteq \{7, 31, 19, 93\}$

Adder 3:

- $7 = c_{3,l} = c_{ind_{3,l}} = c_1$
  $31 = c_{3,r} = c_{ind_{3,r}} = c_2$

a	$ind_{a,l}$	$ind_{a,r}$
1	0	0
2	0	0
3	1	2
4	2	2

Constraints - Reducing space search

Dempster theorems (1994) [Dempster, 1994]:

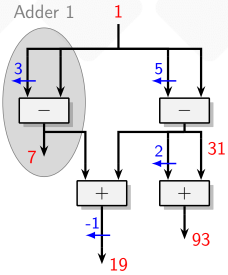
- Positiveness of fundamentals
- Left shift only on the left input or (exclusive) right shift on both inputs
- Oddness of fundamentals

Constraints:

C8: $k_{a,l} > 0 \ \vee \ k_{a,r} > 0$

C9: $k_a + k_{a,l} \neq 2$ and 0

C10: $c_a \equiv 1 (\mod 2)$



Adder 1:

- $k_{1,l} = (-1)^0 2^3 = 8 > 0, \ k_{1,r} = (-1)^1 2^0 = -1 < 0$
- $k_1 + k_{1,l} = 2^0 + (-1)^0 2^3 = 1 + 8 = 9 \neq 0$ and 2
- $c_1 = 7 \equiv 1 (mode\ 2)$

Constraints - Free adders

Definition (Free adders)

Free adders are not connected to any other adder.

Theorem (Free target constant)

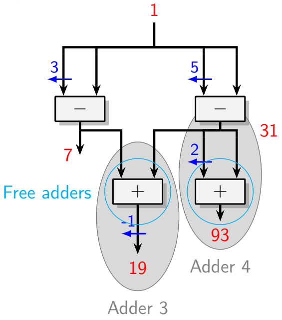
In optimal adder graphs, $\forall\ a \in [\![1, N]\!]$,

a is a free adder $\Rightarrow$

c_a is a target constant

Constraint:

C11: $(\nexists\ a' \in [\![a+1, N]\!] \mid ind_{a',l} = a\ \vee\ ind_{a',r} = a) \implies c_a \in T$



- 3 is a free adder $\Rightarrow c_3 = 19 \in \{7, 19, 93\} = T$
- 4 is a free adder $\Rightarrow c_4 = 93 \in \{7, 19, 93\} = T$

CP model - MCM problem

Data and variables
$N \in \mathbb{N}$: adders number; $w \in \mathbb{N}$: fundamentals word length; $T \subset \mathbb{N}$: target constants set;
$\forall\ a \in [\![0, N]\!],\ c_a \in [\![1, 2^w - 1]\!]$: fundamental of adder a with $c_{\mathbf{0}} = 1$; $\forall\ a \in [\![1, N]\!],\ c_a^{nsh} \in [\![0, 2^{w+\mathbf{1}}]\!]$: value of adder a before negative shift; $\forall\ a \in [\![1, N]\!],\ c_{a,l},\ c_{a,r} \in [\![1, 2^w - 1]\!]$: value of input i of adder a; $\forall\ a \in [\![1, N]\!],\ c_{a,l}^{sh,sg},\ c_{a,r}^{sh,sg} \in [\![-2^{w+\mathbf{1}}, 2^{w+\mathbf{1}}]\!]$: signed / shifted value from left or right input of adder a; $\forall\ a \in [\![1, N]\!],\ k_{a,l} \in \{-2^s, 2^s \mid s \in [\![0, w]\!]\}$: value of sign and left shift applied to $c_{a,l}$; $\forall\ a \in [\![1, N]\!],\ k_{a,r} \in \{-1, 1\}$: value of sign applied to $c_{a,r}$; $\forall\ a \in [\![1, N]\!],\ k_a \in \{-2^s, 2^s \mid s \in [\![0, w]\!]\}$: value of right shift applied to c_a^{nsh}; $\forall\ a \in [\![1, N]\!],\ ind_{a,l},\ ind_{a,r} \in [\![0, a-1]\!]$: index of left or right input of adder a; $\forall\ a \in [\![1, N]\!],\ l_a \in [\![0, N-a]\!]$: number of adder using c_a as an input;

(a) Data / Variables of the CP model and their meaning

C1: $\forall\ a \in [\![1, N]\!],\ k_a \times c_a = c_a^{nsh}$

C2: $\forall\ a \in [\![1, N]\!],\ \begin{cases} k_{a,l} \times c_{a,l} = c_{a,l}^{sh,sg} \\ k_{a,r} \times c_{a,r} = c_{a,r}^{sh,sg} \end{cases}$

C3: $\forall\ a \in [\![1, N]\!],\ c_{a,l}^{sh,sg} + c_{a,r}^{sh,sg} = c_a^{nsh}$

C4: $\forall\ a \in [\![1, N]\!],\ c_{a,l} = c_{ind_{a,l}} \wedge c_{a,r} = c_{ind_{a,r}}$

C5: $T \subseteq \{c_a \mid a \in [\![1, N]\!]\}$

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C8: $\forall\ a \in [\![1, N]\!],\ k_{a,l} > 0 \vee k_{a,r} > 0$

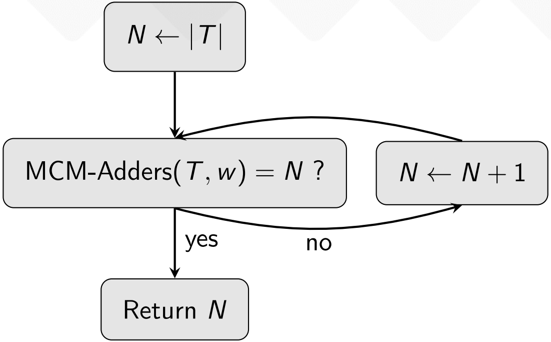
C9: $\forall\ a \in [\![1, N]\!],\ \begin{cases} k_a + k_{a,l} \neq 2 \\ k_a + k_{a,l} \neq 0 \end{cases}$

C10: $\forall\ a \in [\![1, N]\!],\ c_a \equiv 1 (mode\ 2)$

C11: $\forall\ a \in [\![1, N]\!],\ (\nexists\ a' \in [\![a+1, N]\!] \mid ind_{a',l} = a \vee ind_{a',r} = a) \implies c_a \in T$

(b) CP model for the Decision problem, when the number of adders is fixed

From decision to optimization



Outer loop process

2 types of minimization:

- Outer loop: Transforming minimization into a succession of decision problems
- Minimization: Integrating the minimization into an objective function

Implementations and experiments

Our implementations:

- CP – Choco: outer-loop CP model implemented in Java with Choco-solver
- CP – PicatSAT: outer-loop model implemented in MiniZinc (CP) + PicatSAT + KisSAT

Comparisons with:

- ILP – [Garcia, 2023]: minimization ILP model implemented in Julia with Gurobi
- SAT – [Fiege, 2024]: outer-loop SAT model implemented in C++ with Cadical

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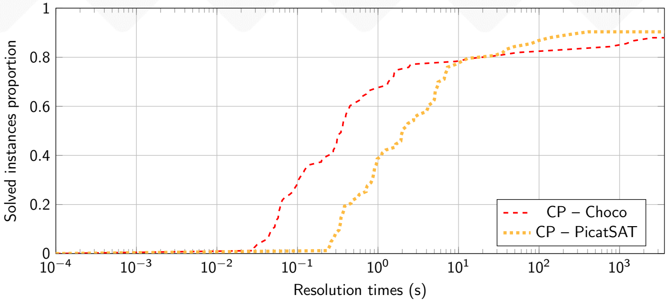
- Experiments on Intel® Xeon® Gold 6444Y (16 cores, 32 threads, 45 MB L3 cache) with 256 GB RAM
- Widely-used benchmark extracted from a collection of linear digital filters that can be implemented with MCM [Nanyang Technological University, 2023]:
  - 83 instances
  - $1 \leq |T| \leq 51$
  - $4 \leq w \leq 19$

Results

Performance results - MCM problem

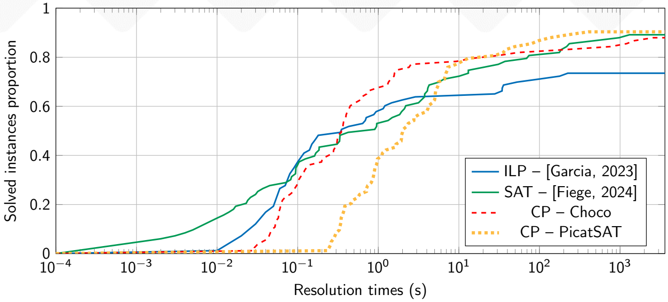


Distribution of the solved instances proportion through time

- Time-out set to 1h

Type	Solved
CP – Choco	88%
CP – PicatSAT	90%

Performance results - MCM problem



Distribution of the solved instances proportion through time

- Time-out set to 1h

Type	Solved
CP – Choco	88%
CP – PicatSAT	90%
ILP – [Garcia, 2023]	68%
SAT – [Fiege, 2024]	89%

Pseudo-polynomial-time preprocessing step

Problem: T-OPT

Input: Target constant set T and word-length w

Question: Do we have MCM-Adders$(T, w) = |T|$?

Theorem (T-OPT)

T-OPT can be solved with a pseudo-polynomial algorithm in $O(|T|^3 \log(w))$ (proof in paper).

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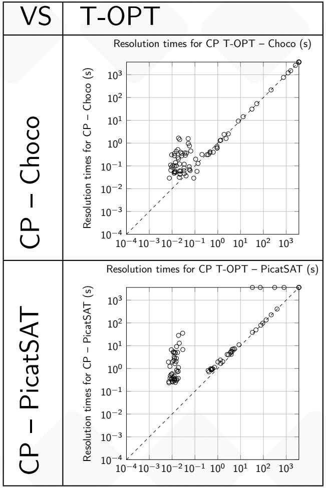
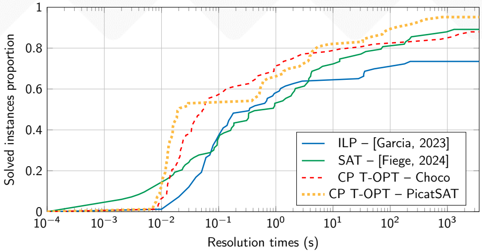
Consequence $\Rightarrow$ preprocessing step:

- If T-OPT$(T, w) = \mathit{true}$, then stop.
- If T-OPT$(T, w) = \mathit{false}$, then launch CP model with constraint MCM-Adders$(T, w) > |T|$.

T-OPT is true for 44 out of the 83 MCM instances of the benchmark!

Performance results with preprocessing step - MCM problem





Distribution of the solved instances proportion through time

Our models are very competitive with state-of-the-art!

Conclusion

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Takeaway

- On many combinatorial problems, modeling with ILP is very fastidious in comparison with CP
- Very constrained specific cases lead to easy resolution

Further work and perspectives

- Fixed adder graph topology MCM problem is easier. Could we test every adder graph topology ?
- Need to improve scalability with respect to the word-length (with $w > 20$ bits)

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- Very constrained specific cases lead to easy resolution

Further work and perspectives

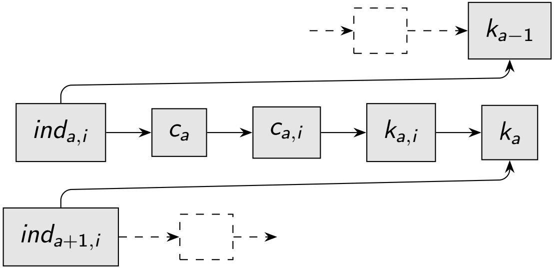
- Fixed adder graph topology MCM problem is easier. Could we test every adder graph topology ?
- Need to improve scalability with respect to the word-length (with $w > 20$ bits)

Thank you for your attention!

Any questions?

Appendix 1 - CP model - Search strategy

- Random exploration of the search space rarely produces valid solutions. Fixing adder 2 before adder 1 $\longrightarrow$ failure
- Solution: constructing the adder graph incrementally
- Lower Bound First strategy



Appendix 2 - ILP model - MCM problem

Constants/Variables and their meaning
$N \in \mathbb{N}$: number of adders; $C \in \mathbb{N}^{N_O}$: set of odd target constants; $w \in \mathbb{N}$: fundamentals' word length;
$c_a \in [\![0; 2^w]\!]$, $\forall a \in [\![0; N]\!]$: fundamental, or constant, obtained in adder a with c_0 fixed to the value 1, corresponding to the input; $c_a^{\text{nsh}} \in [\![0; 2^{w+1}]\!]$, $\forall a \in [\![1; N]\!]$: constant obtained in adder a before the negative shift; $c_a^{\text{odd}} \in \mathbb{N}$, $\forall a \in [\![1; N]\!]$: variable used to ensure that c_a is odd; $c_{a,i} \in [\![0; 2^w]\!]$, $\forall a \in [\![1; N]\!]$, $i \in \{l, r\}$: constant of adder from input i before adder a; $c_{a,l}^{\text{sh}} \in [\![0; 2^{w+1}]\!]$, $\forall a \in [\![1; N]\!]$: constant of adder from left input before adder a and after the left shift; for simplification $c_{a,r}^{\text{sh}}$ is an alias of $c_{a,r}$; $c_{a,i}^{\text{sh,sg}} \in [\![-2^{w+1}; 2^{w+1}]\!]$, $\forall a \in [\![1; N]\!]$, $i \in \{l, r\}$: signed constant of adder from input i before adder a and after the shift; $\sigma_{a,i} \in \{0, 1\}$, $\forall a \in [\![1; N]\!]$, $i \in \{l, r\}$: sign of i input of adder a. 0 for $+$ and 1 for $-$; $c_{a,i,k} \in \{0, 1\}$, $\forall a \in [\![1; N]\!]$, $i \in \{l, r\}$, $\forall k \in [\![0; a-1]\!]$: 1 if input i of adder a is adder k; $\Phi_{a,s} \in \{0, 1\}$, $\forall a \in [\![1; N]\!]$, $s \in [\![0; w]\!]$: 1 if left shift before adder a is equal to s; $\Psi_{a,s} \in \{0, 1\}$, $\forall a \in [\![1; N]\!]$, $s \in [\![0; w]\!]$: 1 if negative shift of adder a is equal to s; $o_{a,j} \in \{0, 1\}$, $\forall a \in [\![1; N]\!]$, $j \in [\![1; N_O]\!]$: 1 if adder a is equal to the j-th target constant.

(a) Data / Variables of the ILP model and their meaning

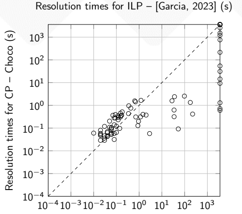
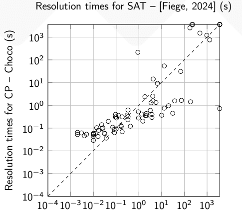
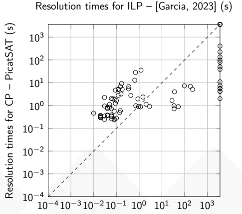
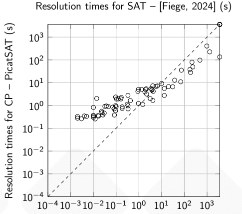
$$c_0 = 1 \qquad \text{(C1.1)}$$
$$c_a^{\text{nsh}} = c_{a,l}^{\text{sh,sg}} + c_{a,r}^{\text{sh,sg}} \qquad \forall a \in [\![1; N]\!] \qquad \text{(C1.2)}$$
$$c_a^{\text{nsh}} = 2^{s_a} c_a \quad \text{if } \Psi_{a,s} = 1 \qquad \forall a \in [\![1; N]\!], s \in [\![0; w]\!] \qquad \text{(C1.3)}$$
$$\sum_{s=0}^{w} \Psi_{a,s} = 1 \qquad \forall a \in [\![1; N]\!] \qquad \text{(C1.4)}$$
$$\Phi_{a,0} + \Psi_{a,s} = 1 \qquad \forall a \in [\![1; N]\!] \qquad \text{(C1.5)}$$
$$c_a = 2c_a^{\text{odd}} + 1 \qquad \forall a \in [\![1; N]\!] \qquad \text{(C1.6)}$$
$$c_{a,i} = c_k \quad \text{if } c_{a,i,k} = 1 \qquad \forall a \in [\![1; N]\!], i \in \{l, r\}, k \in [\![0; a-1]\!] \qquad \text{(C1.7)}$$
$$\sum_{k=0}^{a-1} c_{a,i,k} = 1 \qquad \forall a \in [\![1; N]\!], i \in \{l, r\} \qquad \text{(C1.8)}$$
$$c_{a,l}^{\text{sh}} = 2^s c_{a,l} \quad \text{if } \Phi_{a,s} = 1 \qquad \forall a \in [\![1; N]\!], s \in [\![0; w]\!] \qquad \text{(C1.9)}$$
$$\sum_{s=0}^{w} \Phi_{a,s} = 1 \qquad \forall a \in [\![1; N]\!] \qquad \text{(C1.10)}$$
$$c_{a,i}^{\text{sh,sg}} = -c_{a,i}^{\text{sh}} \quad \text{if } \sigma_{a,i} = 1 \qquad \forall a \in [\![1; N]\!], i \in \{l, r\} \qquad \text{(C1.11)}$$
$$c_{a,i}^{\text{sh,sg}} = c_{a,i}^{\text{sh}} \quad \text{if } \sigma_{a,i} = 0 \qquad \forall a \in [\![1; N]\!], i \in \{l, r\} \qquad \text{(C1.12)}$$
$$\sigma_{a,l} + \sigma_{a,r} \leq 1 \qquad \forall a \in [\![1; N]\!] \qquad \text{(C1.13)}$$
$$c_a = C_j \quad \text{if } o_{a,j} = 1 \qquad \forall a \in [\![0; N]\!], j \in [\![1; |C|]\!] \qquad \text{(C1.14)}$$
$$\sum_{a=0}^{N} o_{a,j} = 1 \qquad \forall j \in [\![1; |C|]\!] \qquad \text{(C1.15)}$$

(b) ILP model for the Decision problem, when the number of adders is fixed to N

Appendix 3 - Comparison with state-of-the-art solvers

	ILP – [Garcia, 2023]	CP – Choco
Minimization	Integrated into the objective function	Outer loop
Intermediate results	Yes	No
Number of variables	$\sim (10 + N + w + \lvert T \rvert)N$	$\sim 12N$
Number of constraints	$\sim (10 + 2N + 4w + 2\lvert T \rvert)N + \lvert T \rvert$	$\sim 13N$

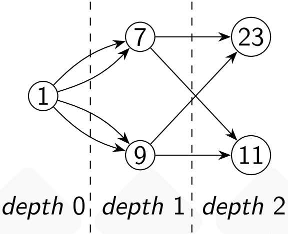
Appendix 4 - Performance results

VS	ILP – [Garcia, 2023]	SAT – [Fiege, 2024]
CP – Choco		
CP – PicatSAT		

Appendix 5 - Polynomial-time preprocessing step

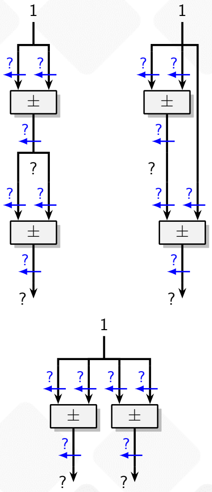
$$\mathrm{can}(x,y,z) =$$

$$\left(\begin{array}{c|l} \exists\ (\sigma_1,\sigma_2) \in & z = odd((-1)^{\sigma_1}x + (-1)^{\sigma_2}y) \\ & \text{or}\ \ (-1)^{\sigma_1}x = odd(z-(-1)^{\sigma_2}y) \\ \{0,1\}^2 \setminus (1,1) & \text{or}\ \ (-1)^{\sigma_2}y = odd(z-(-1)^{\sigma_1}x) \end{array}\right)$$




```
function MCM-Adders_pseudo-poly(T)
    Let R = {1} and W = {1}
    while W ≠ ∅ do
        Pop x from W
        forall z ∈ T \ R do
            forall y ∈ R \ W do
                if can(x, y, z) then
                    prec(z) ← (x, y)
                    R ← R ∪ {z}
                    W ← W ∪ {z}
                    Break
        if R = T then
            return (True, prec)
    return (False, ∅)
```

Appendix 6 - Motivation - Fixing the topology



Interesting results but can we speed up even further computations?

- Fixing the topology make MCM "easier" (still NP-complete)
- **Idea:** testing the MCM problem with fixed topology with every adder graph topology

function MCM-Adders(T)
    Let $n = |T|$
    while *true* **do**
        forall *adder graph G with n vertices* **do**
            if MCM-Adders-FixedTopology(T, w, G) $=$ *true* **then**
                return G
        $n \leftarrow n + 1$

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