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► To cite this version:

Théo Cantaloube, Xiao Peng, Christine Solnon, Anastasia Volkova. A new Constraint Programming model for the Multiple Constant Multiplication. ModRef 2025 - 24th workshop on Constraint Modelling and Reformulation, Aug 2025, Glasgow, United Kingdom. <hal-05251229>

HAL Id: hal-05251229

<https://hal.science/hal-05251229v1>

Submitted on 12 Sep 2025

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A new Constraint Programming model for the Multiple Constant Multiplication

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Abstract

The Multiple Constant Multiplication (MCM) problem arises in many applications such as, for example, digital signal processing. Given a set T of target constants, the goal of MCM is to find the most efficient way for multiplying an input number with each constant in T , where multiplications are realized through bit-shifts and additions, and where intermediate results may be shared to produce different target constants. Different metrics may be considered for evaluating the cost of a solution, and a classical objective function is to minimize the number of adders. State-of-the-art methods, based on Integer Linear Programming (ILP), suffer from numerous performance and scalability bottlenecks. In this work, we propose for the first time a Constraint Programming (CP) model for minimizing the number of adders for the MCM.

Compared to the state-of-the-art ILP approach, CP does not suffer from the curse of linearization, hence permits significantly simpler formulations of the mathematical model. In order to evaluate our CP model, we focus on a widely used benchmark extracted from a collection of digital filter designs and compare ourselves with state-of-the-art ILP and SAT models. We show that our CP approach is less efficient on some easy instances, but more efficient on hard instances. We also introduce a pseudo-polynomial time algorithm which is able to solve some instances, and show that using this algorithm during a preprocessing step improves the solution process.

2012 ACM Subject Classification Theory of computation → Constraint and logic programming

Keywords and phrases Constraint Programming, Multiple Constant Multiplication, Hardware Optimization

Digital Object Identifier 10.4230/LIPIcs...

1 Introduction

1.1 Motivation

Electronic devices with embedded computations are everywhere: in hearing-aids, smart-watches, cars, or drones, for example. The strict resource and power constraints call for highly optimized implementations. Instead of using standard data formats and generic arithmetic units, hardware designers often use code generators to implement individual or compound operators tailored to the application's requirements.

In this context, a common type of arithmetic operator is Multiplication by Multiple Constants (MCM), where an integer number is multiplied by several target constants known at the design time. The idea is that by exploiting the a priori knowledge of the constants, one can avoid costly generic multiplier circuits and design a custom optimized computational implementation for a given target constant set. MCM is used in digital signal processing for



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linear digital filter evaluation [1,13,19,22,26], Discrete Transforms (Discrete Cosine Transform, Fast Fourier Transform) [15,32], as well as for inference of Deep Neural Networks [10,17].

Multiplications by constants are commonly done using additions/subtractions and bit-shifts: shifting a number X by k bits on the left (or the right) is equivalent to multiplying X by 2^k (or 2^{-k}). This is called a *shift-and-add* approach. Shifts are basically wiring that is free in hardware, hence designers usually only minimize the number of additions/subtractions (which we simply refer to as adders). This is a good high-level metric of the final hardware cost, though other low-level ones that count logic gates exist.

Let us first illustrate our problem on a single constant multiplication (SCM) problem. Consider the multiplication of an integer variable X by 93, written as 1011101 in binary. Hence, $93X = 2^6X + 2^4X + 2^3X + 2^2X + X$ and a naive implementation of a shift-and-add approach for $93X$ is:

$$80X = 2^6X + 2^4X; \quad 88X = 80X + 2^3X; \quad 92X = 88X + 2^2X; \quad 93X = 92X + X$$

However, this idea does not take into account the possibility of making subtractions instead of additions. Thanks to the *Canonical Signed Digit* representation of a number [18,31], we obtain that $93X = 2^7X - 2^5X - 2^2X + X$ so a second implementation of a shift-and-add approach for $93X$ is:

$$96X = 2^7X - 2^5X; \quad 92X = 96X - 2^2X; \quad 93X = 92X + X$$

Unfortunately, neither of these approaches are guaranteed to minimize the number of additions. Indeed, the optimal implementation is:

$$31X = 2^5X - X; \quad 93X = 2^1(31X) + 31X$$

You can notice that previous approaches went wrong because they forced every addition to have X as one of the inputs, therefore forbidding additions between two previously computed multiplications.

In MCM, several constants must be multiplied by the same input X . For example, consider the target set of constants $\{7, 19, 31\}$.

A naive implementation of the MCM could consist in optimizing each constant individually:

$$\begin{aligned} 7X &= 2^3X - X \\ 15X &= 2^4X - X; \quad 19X = 15X + 2^2X \\ 31X &= 2^5X - X \end{aligned}$$

However, optimizing each multiplication separately does not ensure that the total number of additions is minimized, since intermediate terms can actually be shared. Indeed, the optimal shift-and-add implementation for this target set is:

$$7X = 2^3X - X; \quad 31X = 2^5X - X; \quad 19X = 2^{-1}(7X + 31X) \quad (1)$$

Note that here we used a right-shift to divide the even value $38X$ by 2 to obtain $19X$.

Given a set of target constants, the MCM Problem aims at finding a shift-and-add implementation that produces every target constant and minimizes a given metric.

There are a lot of MCM non-exclusive variations relying on different metrics used as objective functions to minimize [2,3,12,23]. In this paper, we will focus on the simplest metric: minimizing the number of adders [11,25]. Although it does not perfectly reflect the actual implemented hardware cost, this high-level metric is a good proxy for a cost function.



It is generally accepted that MCM is an $\mathcal{NP}$ -hard problem from [6]. Historically, first papers to address the MCM problem were using heuristics, most of the time based on greedy algorithms [3, 4, 8, 24]. Then Integer Linear Programming (ILP) models were implemented, first by tabulating all possible factors that may occur in an optimal solution in preprocessing computations [3, 16, 20]. However, the size of these number tables grows exponentially with the word-length of target constants, then leading to models requiring highly demanding calculations.

More general new ILP models approached the MCM problem by initializing a lower bound k to the number of target constants, and by iteratively solving the decision problem "Can the MCM be solved with exactly k adders?" until reaching the smallest k for which the answer is yes [21]. This is called an "outer loop process". Instead of solving a sequence of decision problems, authors of [11, 21] introduced a model that directly minimizes the number of adders (while initializing k to an upper bound computed with a good heuristic). Due to the linearization necessity implied by the use of ILP models, these methods suffer from a lack of expressiveness, which translates into really complicated models and also undergo numerical instabilities when the target constants become large.

In the recent paper [9] was proposed a SAT model solving the MCM problem using an outer loop process. Finally, in [5] were introduced SAT models minimizing the number of full/half adders, another lower-level metric, to solve the Single Constant Multiplication Problem. Nevertheless, in this paper, we propose for the first time an approach for the MCM problem from the perspective of Constraint Programming.

1.2 Overview of the paper

In Section 2, we go deeper into some existing models and properties of the MCM problem. In Section 3, we introduce our CP model. In Section 4, we experimentally evaluate our CP model and show that it outperforms the ILP approach of [11] and is competitive with the SAT approach of [9]. In Section 5, we consider a simpler decision problem, that aims at deciding whether there exists a solution when the number of adders is fixed to the number of constant targets. We introduce a pseudo-polynomial time algorithm for this problem, and show that half of the benchmark instances can be solved with this algorithm because the answer is yes. We also show that using this algorithm during a preprocessing step significantly improves the solution process.

2 Problem statement and properties

The first input of an MCM instance is the set T of target constants. Intermediate values used to produce these target constants are called *fundamentals*. Constants that are not used to produce other constants are called *free constants*. For example, in Eq. 1 target constants are 7, 19, and 31. While 19 is a free constant, 7 and 31 are not free as they are used to produce 19.

The second input of an MCM instance is the word-length, *i.e.*, the number of bits w used to encode fundamentals. In this paper, we fix $w = \max_{j \in [1, |T|]} \lceil \log_2(T_j) \rceil$.

We also assume, based on experimentation, that intermediate values of fundamentals can be represented with $w + 1$ bits. Indeed, optimal solutions can often pass by a fundamental that is larger than the largest target constant. For example, in Eq. 1, the target set is representable with 5 bits but the last adder requires 6 to represent the fundamental $7 + 31 = 38 > 31 = 2^5 - 1$.



2.1 Adder modeling

For each adder a , we note $c_{a,l}$, $c_{a,r}$, and c_a the left operand, the right operand, and the result. As each operand is shifted before being added, a first possibility is to use two shift variables for representing the number of bit-shifts that must be performed on the left and right operands, respectively. As shifts may either be to the left (to multiply by positive powers of 2) or to the right (to multiply by negative powers of 2, i.e. divide), these variables are assigned to positive values when the shift is to the left, and to negative values when it is to the right.

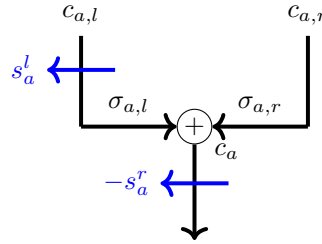
We can reduce the domain of shift variables by considering a theorem of [8] according to which we can impose that either the shift on the left operand is positive and there is no shift on the right operand, or both shifts are negative and equal, so that they can be applied after the addition. Then we obtain [11]:

$$c_a = 2^{-s_a^r} [(-1)^{\sigma_{a,l}} 2^{s_a^l} c_{a,l} + (-1)^{\sigma_{a,r}} c_{a,r}] \quad (2)$$

where s_a^l is the left shift applied to the left input, s_a^r is the right shift applied to the sum of the left and right inputs, and $\sigma_{a,l}$ and $\sigma_{a,r}$ are signs of the left and right inputs. Eq. 2 is equivalent to:

$$2^{s_a^r} c_a = (-1)^{\sigma_{a,l}} 2^{s_a^l} c_{a,l} + (-1)^{\sigma_{a,r}} c_{a,r} \quad (3)$$

Note that $s_a^l, s_a^r \in \mathbb{N}$ instead of $\mathbb{Z}$. These notations are summed up in Fig. 1.



■ **Figure 1** Optimized modeling of adders

► **Example 2.1.** Let us consider the shift-and-add solution Eq. 1. For the adder producing $7 = 2^3 - 1$ we have $c_a = 7$, $c_{a,l} = c_{a,r} = 1$, $s_a^l = 3$, $s_a^r = 0$, $\sigma_{a,l} = 0$ and $\sigma_{a,r} = 1$. For the adder producing $19 = 2^{-1}(31 + 7)$, we have $c_a = 19$, $c_{a,l} = 7$, $c_{a,r} = 31$, $s_a^l = 0$, $s_a^r = 1$, $\sigma_{a,l} = \sigma_{a,r} = 0$.

2.2 Properties of optimal solutions

Here is a basic property of optimal adders:

► **Property 1.** *In optimal implementations, every free constant is a target constant.*

Proof. A free constant is not used to produce other constants. If it is not a target constant, then we can remove it from the circuit. But the implementation is supposed to be optimal, so this is a contradiction. ◀

Keep in mind that the converse is false: a target constant may be used to compute other target constants (for example, in Eq. 1, 7 and 31 are not free constants).

Also, if we denote $\text{MCM-Adders}(T, w)$ the minimal number of adders required to produce the target constants set T on w bits, we have:



162 ► **Property 2.** $MCM\text{-}Adders(T, w) \geq |T|$

163 **Proof.** This is simply because we need an adder output producing each target constant. ◀

164 2.3 Preprocessing of target constants

165 The target constant set T is preprocessed before launching the solving process, as proposed
 166 in [8]. First, we replace every target constant by its absolute value, as the sign may be
 167 adjusted later. Second, we divide every target constant by 2 until it becomes odd, as a shift
 168 can be applied to retrieve the original even-valued constant. Finally, we remove 1 from the
 169 resulting set of target constants, as it can be obtained for free from the initial input. More
 170 formally, T is replaced with $\{odd(|T_j|), T_j \in T\} \setminus \{1\}$ where the operator odd is defined as:

$$171 \quad odd(x) = \frac{x}{\max_{n \geq 0}(gcd(x, 2^n))}$$

172 As a consequence, every preprocessed target constant is odd and greater than 1.

173 3 Constraint Programming model

174 In this section, we first introduce a CP model for the decision problem where the number of
 175 adders is given. Then, we show how to use this model to solve the optimization problem.

176 3.1 Fixed Number of adders Decision Problem

177 In this section, we assume that the number of adders is fixed to N .

178 Variables

179 For each adder $a \in \llbracket 1, N \rrbracket$, we introduce variables to represent its operands and output:

- 180 ■ $c_a \in \llbracket 1, 2^w - 1 \rrbracket$ is the output constant;
- 181 ■ $c_{a,l} \in \llbracket 1, 2^w - 1 \rrbracket$ and $c_{a,r} \in \llbracket 1, 2^w - 1 \rrbracket$ are the left and right inputs.

182 We also introduce a variable c_0 which models the initial input and which is constrained to
 183 be equal to 1.

184 Variables $c_a, c_{a,l}$, and $c_{a,r}$ are related with sign values ($\sigma_{a,l}$ and $\sigma_{a,r}$) and shift values (s_a^l
 185 and s_a^r) as defined in Eq. 3. Instead of introducing sign and shift variables, as done in ILP
 186 models, we gather them in $k_a, k_{a,l}$, and $k_{a,r}$ variables, which are further gathered in c_a^{nsh} ,
 187 $c_{a,l}^{sh,sg}$ and $c_{a,r}^{sh,sg}$ variables as displayed below:

$$188 \quad \underbrace{\underbrace{2^{s_a^r}}_{k_a} c_a}_{c_a^{nsh}} = \underbrace{\underbrace{(-1)^{\sigma_{a,l}} 2^{s_a^l}}_{k_{a,l}} c_{a,l}}_{c_{a,l}^{sh,sg}} + \underbrace{\underbrace{(-1)^{\sigma_{a,r}}}_{k_{a,r}} c_{a,r}}_{c_{a,r}^{sh,sg}} \quad (4)$$

189 Hence, for each adder $a \in \llbracket 1, N \rrbracket$, we introduce the following variables to store intermediate
 190 results:

- 191 ■ $k_a \in \{2^s \mid s \in \llbracket 0, w \rrbracket\}$;
- 192 ■ $k_{a,l} \in \{-2^s, 2^s \mid s \in \llbracket 0, w \rrbracket\}$;
- 193 ■ $k_{a,r} \in \{-1, 1\}$;
- 194 ■ $c_a^{nsh} \in \llbracket 0, 2^{w+1} \rrbracket$;
- 195 ■ $c_{a,l}^{sh,sg} \in \llbracket -2^{w+1}, 2^{w+1} \rrbracket$ and $c_{a,r}^{sh,sg} \in \llbracket -2^w + 1, 2^w - 1 \rrbracket$.



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- C1:** $\forall a \in \llbracket 1, N \rrbracket, k_a \times c_a = c_a^{nsh}$
C2: $\forall a \in \llbracket 1, N \rrbracket, k_{a,l} \times c_{a,l} = c_{a,l}^{sh,sg} \quad \text{and} \quad k_{a,r} \times c_{a,r} = c_{a,r}^{sh,sg}$
C3: $\forall a \in \llbracket 1, N \rrbracket, c_{a,l}^{sh,sg} + c_{a,r}^{sh,sg} = c_a^{nsh}$
C4: $\forall a \in \llbracket 1, N \rrbracket, k_{a,l} > 0 \quad \text{or} \quad k_{a,r} > 0$
C5: $\forall a \in \llbracket 1, N \rrbracket, k_a + k_{a,l} \neq 2 \quad \text{and} \quad k_a + k_{a,r} \neq 0$
C6: $T \subseteq \{c_a \mid a \in \llbracket 1, N \rrbracket\}$
C7: $c_0 = 1$
C8: $\forall a \in \llbracket 1, N-1 \rrbracket, \{a' \in \llbracket k+1, N \rrbracket \mid ind_{a',l} = a \vee ind_{a',r} = a\} = \emptyset \implies c_a \in T$
C9: $\forall a \in \llbracket 1, N \rrbracket, c_{a,l} = c_{ind_{a,l}} \quad \text{and} \quad c_{a,r} = c_{ind_{a,r}}$
C10: $\text{allDifferent}((c_a)_{a \in \llbracket 0, N \rrbracket})$
C11: $\forall a \in \llbracket 1, N \rrbracket, c_a \equiv 1[2]$

■ **Figure 2** Constraints for the Decision problem, when the number of adders is fixed to N

196 Finally, for each adder $a \in \llbracket 1, N \rrbracket$, we introduce variables for linking the left and right
 197 inputs of a with the output of another adder. To ensure that there is no cycle in this
 198 dependency relation, the domain of these variables only contains values smaller than a :
 199 ■ $ind_{a,l} \in \llbracket 0, a-1 \rrbracket$ and $ind_{a,r} \in \llbracket 0, a-1 \rrbracket$

200 Constraints

201 Constraints are listed in Fig. 2.

202 Constraints C1 to C3 are straightforward consequences of Eq. 4.

203 As fundamentals cannot have negative values, we know that $k_{a,l}$ and $k_{a,r}$ cannot be both
 204 negative. This is expressed by Constraint C4.

205 As stated in Section 2.1, whenever there is no right shift, *i.e.*, $k_a = 1$, then there is a left
 206 shift, *i.e.*, $k_{a,l} \notin \{-1, 1\}$. This is ensured by Constraint C5.

207 To be a valid MCM implementation for the given target set T , adders must produce every
 208 target constant. This is realized by Constraint C6.

209 Constraint C7 initializes the origin input.

210 According to the Property P. 1, free constants are target constants. First member of the
 211 implication of Constraint C8 is equivalent to: "If adder a produces a free constant..." and
 212 second member is equivalent to "...then it is a target constant.".

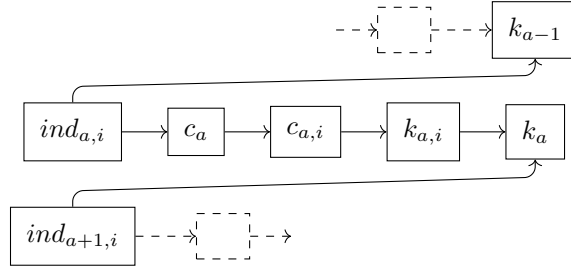
213 Constraint C9 links the left and right inputs of each adder a with the output of adders
 214 $ind_{a,l}$ and $ind_{a,r}$, respectively.

215 Finally, constraints C10 and C11 ensure that the generated values are all different and
 216 odd, respectively.

217 Search strategy

218 Random exploration of the search space in the MCM problem rarely produces valid or
 219 meaningful solutions. Fixing the fundamental of the second adder before the first adder
 220 generally results in failure. In contrast, constructing the adder graph incrementally, by
 221 adding one adder at a time in a well-defined order, is more likely to yield a valid solution.
 222 Thus, we impose a Lower Bound First strategy, and the variable ordering heuristic is detailed
 223 in Fig. 3.





■ **Figure 3** Branching priority for Constraint Programming modeling

3.2 Minimization Problem

To solve the MCM minimization problem, we solve a sequence of decision problems: we initialize N to $|T|$, and then we iteratively solve the model defined in Section 3.1 and increment N until we find a solution.

4 Experimental Evaluation

Our CP model has been implemented in Java with Choco-solver [30]. Experiments were conducted on a machine equipped with an Intel® Xeon® Gold 6444Y processor (16 cores, 32 threads, 45 MB L3 cache) with 256 GB RAM, running on an x86_64 architecture with support for AVX-512 and related vector instruction sets. We call this approach CP – Choco.

In addition to CP – Choco, we propose one more model using Minizinc [28]. As we heard how efficient SAT solvers (directly written in SAT) were for the MCM problem resolution, we used Minizinc with CP models in front-end, PicatSAT [33] in middle-end (to translate CP into SAT) and KisSAT in back-end. We denote as CP – PicatSAT our CP model with this approach.

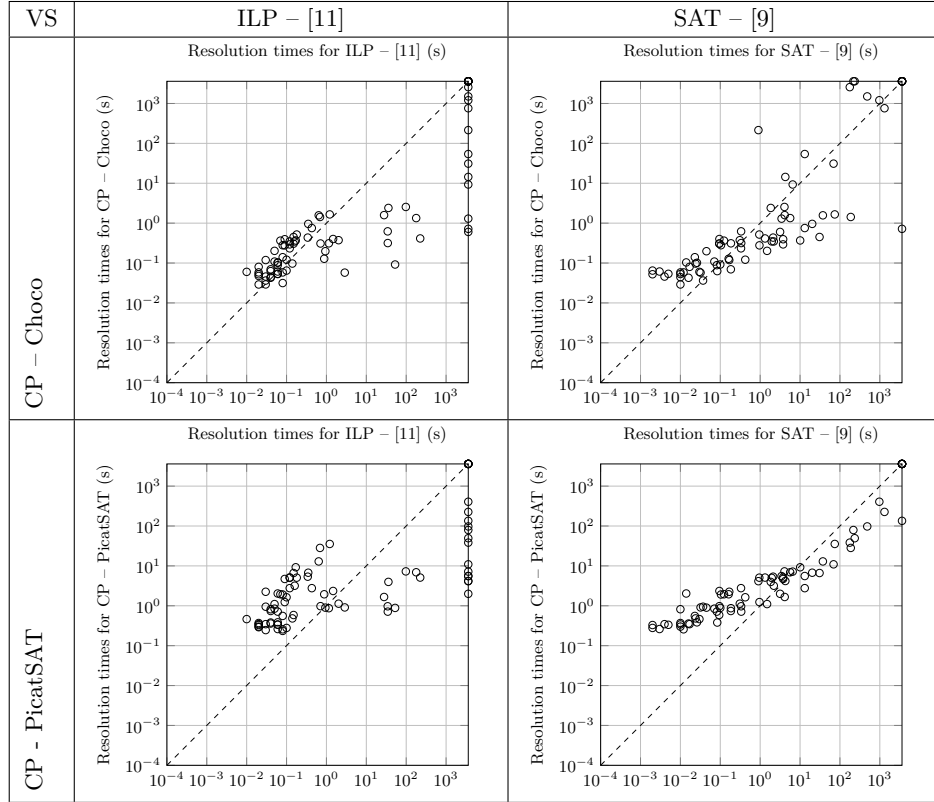
The code written for these models is available as open source¹.

The benchmark we have worked on is extracted from a collection of linear digital filters [27] that can be implemented with the MCM problem. It has been widely used in the literature and for the ILP-based approaches in particular [11, 21, 24]. It contains 83 instances with $|T|$ ranging from 1 to 51 constants and w from 4 to 19 bits.

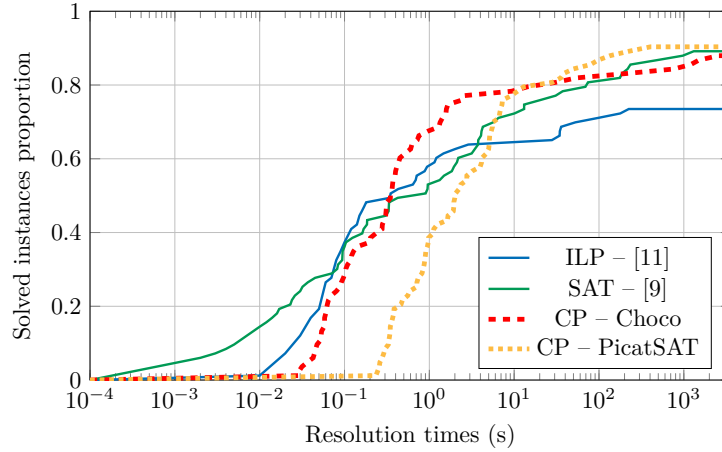
In Fig. 4(a), we compare our CP-based approach with the ILP approach of [11] and the SAT approach of [9] on a per instance basis. ILP – [11] and SAT – [9] are faster on "easy" instances, solved in less than 1s by both approaches. However, our approaches are faster on harder instances. While, in a run time limit of 3600s, CP – Choco and CP – PicatSAT are respectively able to solve 73 and 75 out of 83 instances, ILP – [11] and SAT – [9] respectively solved 61 and 74 out of 83 instances. To give more details, whenever CP – Choco or CP – PicatSAT resulted with a Time-Out, ILP – [11] resulted with a Time-out and whenever CP – PicatSAT resulted with a Time-Out, SAT – [9] resulted with a Time-out. When looking at Fig. 4(b), that displays the evolution of the cumulative percentage of solved instances with respect to time, we note that ILP – [11] and SAT – [9] are more successful than CP – Choco and CP – PicatSAT when the time limit is smaller than 0.1s, whereas SAT – [9], CP – Choco and CP – PicatSAT is more successful than ILP – [11] for longer time limits.

In addition, we can see how much better suited CP – Choco is to the combinatorial

¹ <https://gitlab.inria.fr/emeraude/mcm-cp>



(a) Comparing CP – Choco and CP – PicatSAT to ILP – [11] and SAT – [9]



(b) Distribution of the solved instances proportion through time

■ **Figure 4** Comparative analysis of ILP and CP resolution times.



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aspects of the MCM problem than ILP – [11] by comparing the number of variables in the respective models. ILP – [11] uses $2 + N(10 + |T| + 2w + (N + 1)/2)$ variables whereas CP – Choco uses $11N$ variables. We can notice that unlike ILP – [11], there is no dependence on the number of target constants or on the word-length and the dependence on N is linear.

5 Preprocessing step

In this section, we study the decision problem that aims at deciding whether there exists a solution with $|T|$ adders. We show that this problem may be solved in pseudo-polynomial time, and that the solution process is improved when solving this problem in a preprocessing step.

Let us first formally define this decision problem:

Problem: T-OPT

Input: Target constant set T and word-length w

Question: Do we have $\text{MCM-Adders}(T, w) = |T|$?

Before delving into the algorithm, we first need to show how to efficiently determine if z can be produced with x and y and inputs (we denote this as $\text{can}(x, y, z)$). We could do $\text{can}(x, y, z) = (z \in \mathcal{A}_w(x, y))$ where $\mathcal{A}_w(x, y)$ represents the set of fundamentals written on w bits "reachable" from inputs x and y but computing $\mathcal{A}_w(x, y)$ is not optimal (the table is too large and it takes too much time to generate). Instead, we can use the operator odd and do:

$$\text{can}(x, y, z) = \left(\exists (\sigma_1, \sigma_2) \in \{0, 1\}^2 \setminus (1, 1) \left| \begin{array}{l} z = \text{odd}((-1)^{\sigma_1}x + (-1)^{\sigma_2}y) \\ \text{or } (-1)^{\sigma_1}x = \text{odd}(z - (-1)^{\sigma_2}y) \\ \text{or } (-1)^{\sigma_2}y = \text{odd}(z - (-1)^{\sigma_1}x) \end{array} \right. \right)$$

► **Example 5.1.** Let us use the operator can on $T = \{7, 19, 31\}$ (cf Eq. 1). $\text{can}(1, 1, 7) = \text{true}$ because $1 \times 2^3 - 1 = 7$ so $\text{odd}(7 + 1) = 1$. $\text{can}(1, 1, 31) = \text{true}$ because $1 \times 2^5 - 1 = 31$ so $\text{odd}(31 + 1) = 1$. $\text{can}(7, 31, 19) = \text{true}$ because $2^{-1} \times (7 + 31) = 19$ so $\text{odd}(7 + 31) = 19$.

The time complexity for computing can is logarithmic in the number of bits because of the operator odd [7, chapter 10].

► **Theorem 5.2.** $T\text{-OPT}$ can be solved in pseudo-polynomial time $\mathcal{O}(\log(w) \times |T|^3)$.

Proof. The pseudo-polynomial algorithm is displayed in Alg. 1. We define the depth of an adder as the length of the longest path from the initial input 1, as illustrated in Fig. 5. The idea is to process depth by depth and build R , the set of reached target constants, while Q is the queue of target constants waiting to be processed. We start from 1 and we compute $\text{can}(1, 1, T_j)$, $\forall T_j \in T$. We can repeat the process until we have found all target constants, as illustrated in Fig 5. The algorithm will necessarily terminate because $|T| = \text{MCM-Adders}(T, w)$. The while and for loops on lines 3, 5 and 6 of Alg. 1 run at most $|T|$ times. Hence, the time complexity of Alg. 1 is $\mathcal{O}(\log(w) \times |T|^3)$. ◀

A straightforward consequence of Th. 5.2 is that any instance (T, w) such that $\text{MCM-Adders}(T, w) = |T|$ may be solved in pseudo-polynomial time. Hence, we propose to run Alg. 1 in a preprocessing step. If the algorithm returns true, then prec is a solution and the instance is solved; otherwise we can set the lower bound on the number of adders to $|T| + 1$, and then solve the instance with CP. We call these new approaches CP T-OPT – Choco and CP T-OPT – PicatSAT.



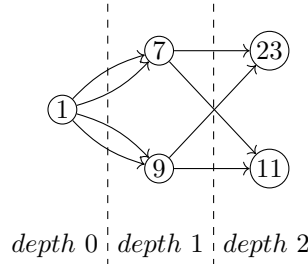
■ **Algorithm 1** Pseudo-polynomial implementation of MCM on T-OPT instances

Input: Target constant set T and word-length w
Output: $(A, prec)$ such that A is the answer for T-OPT(T, w) and if A =true, then $prec$ is a predecessor function corresponding to an MCM implementation

```

1 function MCM-Adderspseudo-poly( $T$ )
2   Let  $R = \{1\}$  and  $W = \{1\}$  be sets of constants
3   while  $W \neq \emptyset$  do
4     Pop  $x$  from  $W$ 
5     forall  $z \in T \setminus R$  do
6       forall  $y \in R \setminus W$  do
7         if  $\text{can}(x, y, z)$  then
8            $prec(z) \leftarrow (x, y)$ 
9            $R \leftarrow R \cup \{z\}$ 
10           $W \leftarrow W \cup \{z\}$ 
11          break
12   if  $R = T$  then
13     return ( $\text{True}, prec$ )
14 return ( $\text{False}, \emptyset$ )

```

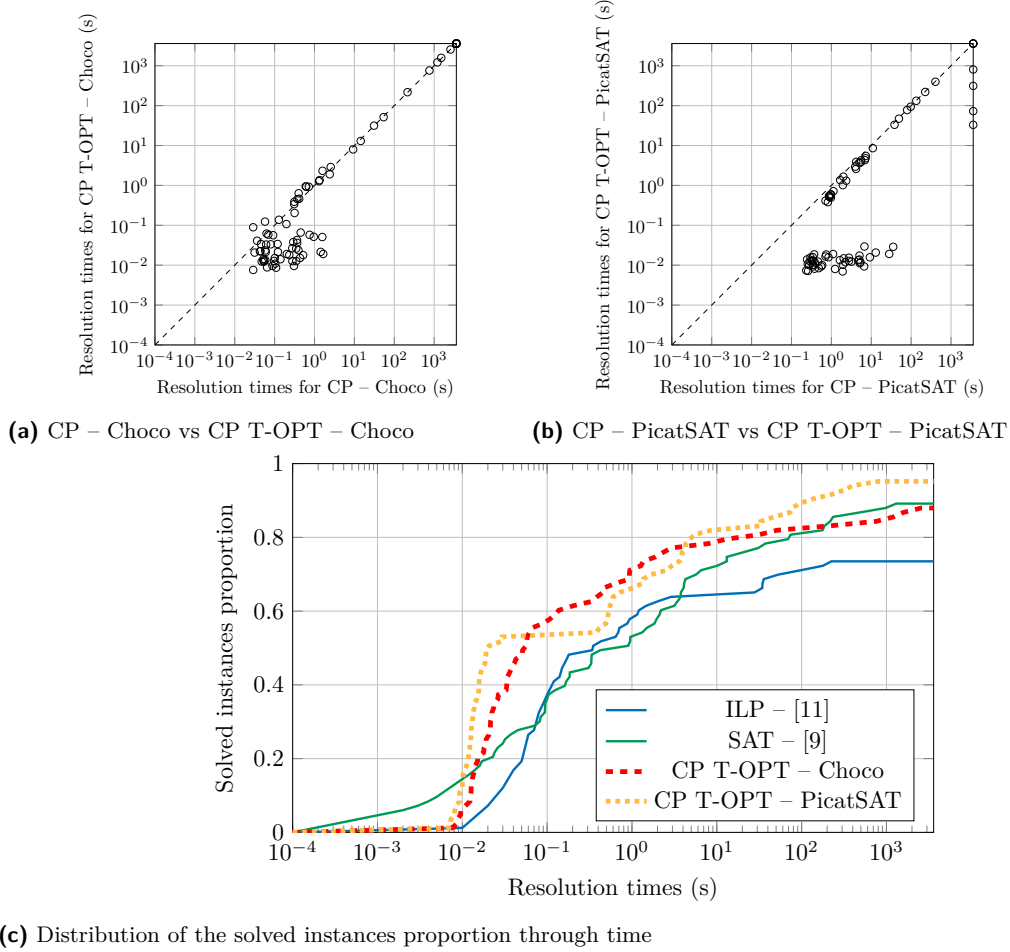


■ **Figure 5** Illustration of Alg. 1 on the toy instance ($T = \{1, 7, 9, 11, 23\}$, $w = 5$). We have $\text{can}(1, 1, 7) = \text{can}(1, 1, 9) = \text{true}$ because $7 = 2^3 - 1$ and $9 = 2^3 + 1$. Hence, at the end of the first iteration of the while loop, we have $R = \{1, 7, 9\}$ and $W = \{7, 9\}$. We have $\text{can}(7, 9, 11) = \text{can}(7, 9, 23) = \text{true}$ because $11 = 2 \times 9 - 7$ and $23 = 2 \times 7 + 9$. Hence, at the end of the second iteration of the while loop, we have $R = \{1, 7, 9, 11, 23\}$ and $W = \{23, 11\}$. Then $R = T$ so the algorithm will return $prec = \{7 \leftarrow (1, 1), 9 \leftarrow (1, 1), 11 \leftarrow (7, 9), 23 \leftarrow (7, 9)\}$

295 When running Alg. 1 on our benchmark, we noticed that the answer was true for 44 out
296 of the 83 MCM instances. In Fig. 6(a) and (b), we compare CP T-OPT – Choco with CP –
297 Choco and CP T-OPT – PicatSAT with CP – PicatSAT. It shows us that the preprocessing
298 step allows us to significantly reduce the solution time for many instances, those for which
299 $\text{MCM-Adders}(T, w) = |T|$, whereas solving times are not significantly changed for the other
300 instances (as the running time of Algo. 1 is largely negligible compared to the running time
301 of CP). When looking at Fig. 6(c), that displays the evolution of the cumulative percentage
302 of solved instances with respect to time, we note that CP T-OPT – Choco and CP T-OPT
303 – PicatSAT are always better than ILP – [11] and SAT – [9]. CP T-OPT – PicatSAT is
304 now able to solve 79 out of 83 instances of the benchmark. This improvement is a direct
305 consequence of the preprocessing step: by avoiding unnecessary resolution of the decision



306 problem when the number of adders equals the number of target constants, more time
 307 remains available to solve harder instances before reaching the timeout.



■ **Figure 6** Comparative analysis of ILP – [11], SAT – [9], CP T-OPT – Choco and CP T-OPT – PicatSAT resolution times.

308 6 Conclusion and perspectives

309 In this paper, we introduced a new Constraint Programming model to solve the Multiple
 310 Constant Multiplication problem. We demonstrated that it outperforms previous ILP-based
 311 approaches and is very competitive in comparison to SAT models. We have also shown
 312 that half of the usual benchmark MCM instances do not require modeling and optimization
 313 paradigms but only a pseudo-polynomial algorithm.

314 Further works will focus on the improvement of the scalability with respect to the word-
 315 length (with $w > 20$ bits). Moreover, it seems that the demonstration of $\mathcal{NP}$ -hardness of
 316 the MCM problem from [6] is not correct because it assumes that the MCM problem is the
 317 same as the Ensemble Computation problem proven $\mathcal{NP}$ -hard in [14], which is false. Thus,
 318 a proper demonstration of the $\mathcal{NP}$ -hardness of the MCM problem would be useful.



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We also plan to extend our model so that it directly solves the minimization problem, instead of solving a sequence of decision problems with fixed numbers of adders. The key point is to model the fact that some adders are not used. This should allow us to improve performance of CP.

Finally, we plan to approach the MCM problem through the prism of fixed topology and decision problems. The idea is that fixing the topology of the shift-and-add implementation (the adder nodes and the arcs) facilitates the search for the corresponding shifts, signs and fundamentals. Hence, we plan to enumerate every graph topology up to a reasonable size to facilitate the optimization. In order to eliminate symmetries we plan to investigate the interest of using canonical codes based on Breadth-First Search, as proposed in [29].

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